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LINEWIDTH OF GAIN-COUPLED DISTRIBUTED FEEDBACK LASERS

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Abstract

A simple expression for the linewidth of vertical-cavity periodic-gain laser diodes is reported. Quantum wells, spaced half-a-wavelength apart are assumed to be driven by time-independent electrical currents J_k, and to possess distinct phase-amplitude coupling factors α_k. The usual linewidth formula is found to be applicable if the photon lifetime is defined as the round-trip time divided by a factor of 6, and the averaging of the α_k is made with J_k as a weighting factor. Furthermore, half the spatial variance of α must be subtracted. The expression involves cross terms of the form $\alpha_i \alpha_j$, $i \neq j$.

1 INTRODUCTION

Conventional Fabry-Pérot laser diodes tend to oscillate on many longitudinal modes. The introduction of a periodic gain leads to drastic improvements. This concept has been implemented for laser diodes with light propagating along the junction plane [1] and with light propagating perpendicular to the junction plane ("vertical-cavity surface-emitting lasers": VCSEL) [2]. In the first case, the coupled-mode theory presented by Kogelnik and Shank and later refined [3] is most appropriate.

In the present paper we consider mainly VCSELs. The thin gain regions (perhaps Quantum wells) are located at the peaks of the optical field [4]. They are accurately modeled by admittances periodically connected to a transmission line [5]. The optical feedback is entirely due to reflection from the gain regions. Because gain and reflection from individual wells are small, such a configuration requires many wells, perhaps 1000. For half-wavelength well-spacings, this corresponds to a total length L = 220 μm at 1.55 μm, assuming an index of refraction of 3.5. In the model treated, the left-hand-side is fully reflecting, while the right-hand-side (optical output) is antireflection-coated. The effective photon lifetime pertaining to modulation and noise is found to be one sixth of the round-trip time. This small value implies large modulation bandwidths, but, in counterpart, rather large spectral widths.

The linewidth is importantly affected by electrical driving conditions, namely, the driving impedance and the injected electrical noise, particularly when the active layers have different phase-amplitude coupling factors [6]. To our knowledge, linewidth formulas that takes the electrical drive into consideration have not been reported. The introduction of a longitudinal K-factor seems to be accurate only in the linear regime.

Idealized configurations involving n active layers driven by time-independent currents J_k, not all necessarily equal, are considered. This driving condition is plausible when the current is injected from the side as shown in Fig.(1a) through high-resistivity layers. Indeed, Nyquist's noise currents associated with high resistances at room temperature is

negligible compared to shot-noise [7]. The phase-amplitude coupling factors α_k and the population inversion factors n_{pk} need not be the same for different layers. For simplicity, we restrict ourselves to the much-above-threshold operation and neglect spontaneous carrier recombination in comparison with stimulated recombination. Our conclusion is that the effective phase-amplitude coupling factor should be averaged with the injected current as a weighting factor. One must further subtract half the spatial variance of α, as found earlier for ring-type configurations [6]. Spectral-hole burning [8, 9] is briefly discussed.

2 STEADY-STATE OPERATION

The gain-grating laser is modeled by n admittances Y_k connected to a transmission line at z = kp, where k = 1,..., n. The period p equals half a wavelength at the operating frequency ν_o. The left-hand side of the transmission line is open circuited, while the right-hand side constitutes the matched output port, see Fig.1b. For simplicity, and without loss of generality, the characteristic conductance of the transmission line is taken as unity.

It is well-known that when a transmission line is loaded by some admittance Y at z = 0, the ratio I/V equals - Y at every half wavelength. For later convenience, we let $V\sqrt{2h\nu_o}$ and $I\sqrt{2h\nu_o}$, rather than V and I, denote the voltage and current, respectively, along the transmission line. I is positive if directed along the z-axis. The photonic rate R, defined as the electromagnetic power divided by the photon energy, is the real part of V*I.

The steady-state oscillation condition is

$$Y_1 + \ldots + Y_n + 1 = 0 \tag{1a}$$

whose real and imaginary parts are

$$G_1 + \ldots + G_n + 1 = 0, \qquad B_1 + \ldots + B_n = 0 \tag{1b}$$

setting for any admittance: $Y \equiv G + iB$.

When the active regions (e.g., quantum wells) are electrically driven from the side as shown in Fig. 1a, the admittances may be nearly equal: $G_k = - 1/n$ and $B_k = 0$ for all k, the power gain per active layer being approximately equal to $1+1/n$. For conventional Fabry-Pérot laser diodes of equal length, this gain value would imply mirror reflectivities: $R_1 = 1$, $R_2 \approx 0.135$, corresponding to photon lifetimes half the round-trip time. In the present situation, feedback is provided by the active layers themselves.

The steady-state values of the admittances are assumed real (pure gain-grating) but not necessarily equal. Deviations of the total admittance from its (zero) steady-state value caused by frequency and carrier number deviations are evaluated in Sections 3 and 4, respectively. The instantaneous frequency deviation $\delta\nu$ follows by specifying that the total admittance remains equal to zero. Upper bars denoting steady-state values are omitted when no confusion with instantaneous values may arise.

3 DISPERSION

Taking it for granted that the admittances Y_k satisfy Eq.(1) at the center frequency ν_o, the admittance at $z = L$ is evaluated for a deviation $\delta\nu$ of the optical frequency.

The equations of propagation along a uniform transmission line of characteristic admittance unity are

$$V(z) = V_+ \exp(ikz) + V_- \exp(-ikz) \tag{2a}$$

$$I(z) = V_+ \exp(ikz) - V_- \exp(-ikz) \tag{2b}$$

where V_+ and V_- are complex constants and

$$k = 2\pi \nu n_r / c = 2\pi/\lambda \tag{3}$$

where n_r denotes the refractive index, c the speed of light in free space, and λ the wavelength in the transmission medium.

At $z = 0$

$$Y_1 = - I(0) / V(0) = - (V_+ - V_-)/(V_+ + V_-) \tag{4}$$

The admittance just before $z = p$ is

$$Y(p_-) \equiv - I(p_-) / V(p_-) = - (V_+ e^{i\phi} - V_- e^{-i\phi})/(V_+ e^{i\phi} + V_- e^{-i\phi}) \tag{5}$$

where

$$\phi = k\, p = k_o p + \delta\phi = \pi + \delta\phi \tag{6a}$$

$$\delta\phi = 2\pi\, \tau^\circ\, \delta\nu, \qquad \tau^\circ = p/v_g, \quad v_g = 2\pi\, d\nu/dk \tag{6b}$$

since p equals a half-wavelength at the center frequency ν_o. The group index c/v_g is of the order of 4 for most III-V coumpounds.

Expending the right-hand-side of Eq.(5) we obtain to first order in $\delta\phi$

$$Y(p_-) \approx (Y_1 + i\delta\phi)/(1 + i\delta\phi\, Y_1) \approx Y_1 + i\, \delta\phi\, (1 - Y_1^2) \tag{7}$$

Proceeding to the next period, Y_2 is added to $Y(p_-)$, and the process in Eq.(7) is iterated. Keeping only terms of order $\delta\phi$, the total admittance $\delta Y_{(\nu)}$ at $z = n\, p \equiv L$ (the subscript ν is a reminder that the change of Y results from frequency deviation) is found to be

$$\delta Y_{(\nu)} = i\, \delta\phi\, [\, n - 1 - Y_1^2 - (Y_1 + Y_2)^2 - ... - (Y_1 + ... + Y_{n-1})^2] \tag{8}$$

the terms in bracket being steady-state values. The expression in Eq.(8) in fact holds at any $z = k\, p$.

When the admittances are equal to $- 1/n$, Eq.(8) reads

$$\delta Y_v = i\,\delta\phi\,\{n - 1 - [1 + 2^2 + \ldots + (n-1)^2] / n^2\} \tag{9a}$$

Using Eq.(6b) and a well-known summation formula, Eq.(9a) reads

$$\delta B_v = 4\pi\,\tau_p\,\delta\nu \tag{9b}$$

$$\tau_p/\tau \equiv (4\,n^2 - 3\,n - 1)/(24\,n^2) \tag{9c}$$

where the round-trip time $\tau \equiv 2\,n\,\tau^o$. Anticipating the final result of this paper, we have introduced in Eq.(9) a photon lifetime τ_p that vanishes as expected for n = 1 and is equal to $\tau/6$ in the large n limit.

The voltage moduli $|V_k|$ is independent of k to first-order in $\delta\phi$ when the admittances Y_k are real in the steady-state. Indeed, we have from Eqs.(2) and (6)

$$V_1 = V_+ + V_- \qquad I_1 = V_+ - V_- = - Y_1 V_1 \tag{10a}$$

$$- V_2 = V_+ e^{i\delta\phi} + V_- e^{-i\delta\phi} \approx (V_+ + V_-) + (V_+ - V_-)\, i\,\delta\phi$$

$$= V_1\,(1 - Y_1\, i\delta\phi) \approx V_1\,(1 - G_1\, i\delta\phi) \tag{10b}$$

Hence

$$|V_2| = |V_1| \tag{10c}$$

to first order. It follows by iteration that $|V_k| = |V_1|$ for all k. The voltage $V \equiv V_n$ is henceforth considered real, without loss of generality.

4 CARRIER FLUCTUATIONS AND NOISE

Deviations of an admittance from its steady-state value as a result of carrier-number variation and noise are now considered, the sign convention being shown in Fig.1c. Nyquist's-like noise sources are

represented in the narrow-band approximation by complex currents $c(t)$ [8]. Accordingly

$$I = Y\,V + c, \qquad c = c' + i\,c'' \tag{11}$$

c' and c'' are independent, and their (double-sided) spectral densities are

$$S_{c'} = S_{c''} = \eta\,G, \qquad \eta \equiv (N_1 - N_2)/(N_1 + N_2) \equiv 1 - 2\,n_p \tag{12}$$

where N_1 and N_2 denote the number of atoms in the lower and upper states, respectively. The population inversion (or spontaneous emission) factor n_p has been introduced. For active admittances with complete population inversion $\eta = -1$ and n_p is unity. At room temperature n_p is of the order of 2.

The discussion is restricted to high-impedance electrical drives. In that situation the injected rates do not fluctuate [7]. The photon generation rate does not fluctuate either at low frequencies since spontaneous carrier recombination is neglected. In the present model all the generated photons are absorbed in a single load and thus the aborbed rate does not fluctuate.

For a cold absorber (subscript o) such as the matched load terminating the transmission line in Fig.(1b), G =1 and η = 1. Because G is a constant, the deviation of I/V from the steady-state value results only from the noise term c_o (sometimes refered to as "vacuum fluctuation")

$$\delta Y_{to} \equiv \delta(I/V) = c_o/V \;\Rightarrow\; V\,\delta G_{to} = c'_o, \quad V\,\delta B_{to} = c''_o \tag{13}$$

where the subscript "t" for "total" indicates that the noise term is included.

Since the absorbed photonic rate R does not fluctuate, Eq.(11) with V real leads to

$$\delta[\mathrm{Re}(V^*I)] = \delta P + V\,c'_o = 0, \qquad P \equiv |V|^2 \;\Rightarrow\; \delta P = - V\,c'_o \tag{14a}$$

We established earlier that the $P_k \equiv |V_k|^2$ are all equal at the center frequency independently of the Y_k values, and remain so under small frequency changes. Accordingly, Eq.(14a) predicts that

$$\delta P_k = \delta P = - V\, c'_o \tag{14b}$$

Consider next an active element (G < 0) with constant injected electronic rate. The output photonic rate does not vary, but G varies as a result of carrier-number variations. Using Eq. (11), the generalization of Eq.(14a) is

$$\delta[\mathrm{Re}(V^*I)] = G\,\delta P + P\,\delta G + V\,c' = 0, \qquad P \equiv |V|^2 \tag{15}$$

and the total admittance change is

$$\delta Y_t \equiv \delta(I/V) = (1 + i\,\alpha)\,\delta G + c\,/\,V \tag{16a}$$

where

$$\alpha \equiv B_N/G_N \tag{16b}$$

is Lax's phase-amplitude coupling factor [10]. In Eq. (16b) the N subscripts denote derivatives with respect to the carrier number N, at some optical frequency. This definition of α implies that Y depends only on N, as is the case when thermal equilibrium is established within the conduction and valence bands.

Substituting the expression for δG in Eq.(15) into Eq.(16a) we obtain

$$\delta Y_t = - (1 + i\alpha)\,(G\,\delta P/P + c'/V) + c/V \tag{17a}$$

whose imaginary part is

$$\delta B_t = - \alpha\,(G\,\delta P/P + c'/V) + c''/V \tag{17b}$$

Consider now the specific configuration in Fig. 1b. When the expression for δP in Eq.(14) is substituted into Eq.(17b), and subscripts k are appended, we obtain

$$V\,\delta B_{tk} = c''_k + \alpha_k\,(\,G_k\,c'_o - c'_k) \tag{18}$$

Let $\delta B_{(N)}$ (subscript N being a reminder that the change is due to carrier number and noise) denote the total change of susceptance, i.e., the sum of the δB_{tk} in Eq.(18) and δB_o in Eq.(13)

$$V\,\delta B_{(N)} = c''_o + \Sigma\,[c''_k + \alpha_k\,(\,G_k\,c'_o - c'_k)] \tag{19}$$

where the sum is from k = 1 to n.

We are now in position to give a general expression for the laser linewidth.

5 LINEWIDTH FORMULA

The instantaneous frequency deviation $\delta\nu$ is obtained by specifying that

$$\delta B_{(\nu)} + \delta B_{(N)} = 0 \tag{20}$$

where δB_ν is given in Eq.(9) and δB_N in Eq.(19):

$$- 4\pi\,\tau_p\,V\,\delta\nu = c''_o + \Sigma\,[c''_k + \alpha_k\,(\,G_k\,c'_o - c'_k)] \tag{21}$$

The noise terms on the right-hand-side of Eq.(21) are independent. The spectral densities of c'_o and c''_o are both unity, while the spectral densities of c'_k and c''_k are both equal to $\eta_k G_k$. Thus, the spectral density $S_{\delta\nu}$ of $\delta\nu$ is given by

$$(4\pi\, \tau_p\, V)^2\, S_{\delta\nu} = 1 + \Sigma\eta_k G_k + (\Sigma\alpha_k G_k)^2 + \Sigma\alpha_k{}^2\, \eta_k G_k \qquad (21)$$

Since the (full-width at half-power) laser linewidth $\Delta\nu$ is 2π times the (double-sided) spectral density of $\delta\nu$ at low frequencies, we arrive at the linewidth formula

$$2\pi\, J\, \Delta\nu = (1/\tau_p)^2\, [1 + \Sigma\eta_k G_k + (\Sigma\alpha_k G_k)^2 + \Sigma\alpha_k{}^2\, \eta_k G_k\,]/4 \qquad (22)$$

where eJ denotes the total injected current (e is the electron charge). The photon lifetime τ_p is, according to Eq.(9), the round-trip time τ divided by 6.

If we introduce the population inversion factors n_{pk} according to Eq.(12), Eq.(22) reads

$$2\pi\, J\, \Delta\nu = (1/\tau_p)^2\, [< n_p\, (1 + \alpha^2)> - (<\alpha^2> - <\alpha>^2)/2]/2 \qquad (23)$$

The averaging is defined as

$$<x> \equiv \Sigma\, x_k\, J_k\, /\Sigma\, J_k \qquad (24)$$

where the sum is over the active elements, k = 1, ... n, with time-independent injected carrier rates J_k. We have used the fact that $J_k = -\, G_k$ P, P $\equiv |V_k|^2$, for any k.

If, for example, the n_{pk} are unity, the injected currents are equal, and the α_k factors are alternately +5 and -5, the laser linewidth is enhanced with respect to the linear regime (Schawlow-Townes formula) by a factor of 6.75. If the second term in bracket Eq.(23) that results from the supression of injected-current fluctuations were ignored, the factor would be equal to 13 instead.

We have treated explicitly purely gain-coupled distributed feedback configurations, with full reflection on one side and no reflection on the other. It is straighforward to treat by the same method the case of real reflections on both sides, the steady-state oscillation still occurring exactly

at Bragg's wavelength. Of greater difficulty is the case of partly index-coupled gratings because the frequency deviates from Bragg's condition in the steady-state.

At high power spectral-hole burning (SHB) may be significant. The dynamics and noise sources in the diffusion of carriers from the high injection energy to the energy spacing appropriate for interaction with the optical field are, in fact, quite analogous to (spatial) carrier diffusion through resistors. In a circuit model, SHB thus acts as a resistance in series with the external electrical resistance, with negligible added noise (for kT << hν). The optical gain should be considered a function of carrier number and photon rate (rather than photon number), the noise source being at the shot-noise level. An equivallent electrical circuit was presented in [9]. For the laser model presently considered with large driving impedances, SHB should not affect importantly intensity noise. But since SHB unclamps the carrier number and deforms the gain-versus-frequency curve, profound effects on linewidth are expected.

6 CONCLUSION

Lax's laser-oscillator linewidth formula [10] can be adapted to gain-coupled distributed feedback lasers, for an idealized model consisting of gain-layers spaced half-a-wavelength apart, and driven by constant currents. The averaging of the layer phase-amplitude coupling factors (α or α^2) is to be effected with the injected currents as weighting factors. One must substract from the term $1+<\alpha^2>$ half the spatial variance of α [the second term in bracket, Eq.(24)]. The cross-products terms $\alpha_i\, \alpha_j$, $i\neq j$, do not appear in alternative theories that ignore electrical driving conditions. The result in Eq.(23) coincides with the result obtained in [6] for ring-type resonators. The photon lifetime entering the linewidth formula is only one sixth the round-trip time. Broad modulation and spectral linewidths are thus expected, in comparison with Fabry-Pérot laser diodes of equal length.

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FIGURE CAPTION

Figure.1

a) Gain-coupled distributed feedback laser, with light propagating perpendicularly to the layers (i.e. along the z-axis), and current injection is from the side. The gain layers, labeled 1, ... n, spaced half-a-wavelength apart, are represented in b) as admittances connected to a transmission line. The left-hand-side of the transmission line is open circuited (full reflection) while the right-hand-side is matched to an absorbing load (not shown). The sign convention for any admittance and Nyquist's-like current source c is in c).

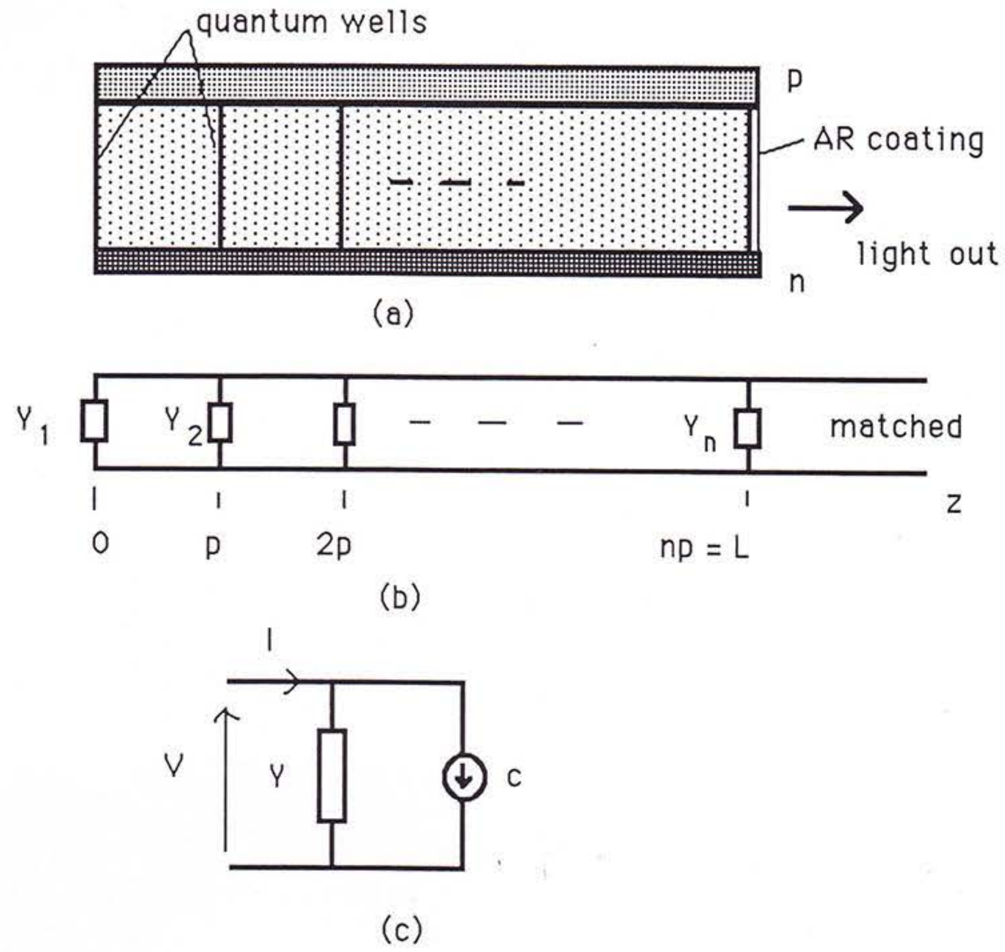

figure I